Econometrics

Causality

Paul P. Momtaz

The Anderson School
UCLA

Outline

The Fundamental Problem with Causal Reasoning

How to deal with it?

Causality in Regressions

Roy (1951) Model

The Fundamental Problem with Causal Reasoning

Notation

- $D_i :=$ treatment for observation i
- $y_i(D_i) :=$ outcome for observation i given treatment

Fundamental Problem (Holland, 1986)

- Impossible to observe for same i the value $D_i = 1$ and $D_i = 0$ as well as $y_i(1)$ and $y_i(0)$.
- That is, there is no true counterfactual evidence.
- Hence it is impossible to observe the effect of D on y for i.

How to deal with it?
ATE and ATT

Average Treatment Effect (ATE)

$$\begin{aligned}\mathbb{E}[\Delta_i] &= \mathbb{E}[y_i(1) - y_i(0)] \\ &= \mathbb{E}[y_i(1)] - \mathbb{E}[y_i(0)]\end{aligned}$$

Average Treatment Effect on the Treated (ATT)

$$\begin{aligned}\mathbb{E}[\Delta_i | D_i = 1] &= \mathbb{E}[y_i(1) - y_i(0) | D_i = 1] \\ &= \mathbb{E}[y_i(1) | D_i = 1] - \mathbb{E}[y_i(0) | D_i = 1]\end{aligned}$$

How to deal with it?
Sample Selection Bias

Sample Selection Bias (SSB)

$$\begin{aligned}
\mathbb{E}[y_i|D_i = 1] - \mathbb{E}[y_i|D_i = 0] &= \mathbb{E}[y_i(1)|D_i = 1] - \mathbb{E}[y_i(0)|D_i = 0] \\
&= \mathbb{E}[y_i(1)|D_i = 1] - \mathbb{E}[y_i(0)|D_i = 1] \\
&+ \mathbb{E}[y_i(0)|D_i = 1] - \mathbb{E}[y_i(0)|D_i = 0] \\
&= \text{ATT} + \mathbb{E}[y_i(0)|D_i = 1] - \mathbb{E}[y_i(0)|D_i = 0] \\
&= \text{ATT} + \text{Sample Selection Bias}
\end{aligned}$$

How to deal with it?

Randomized Experiments

Randomized Experiments:
C, T random samples

$$\mathbb{E}[y_i(0)|i \in C] = \mathbb{E}[y_i(0)|i \in T] = \mathbb{E}[y_i(0)]$$
$$\mathbb{E}[y_i(1)|i \in C] = \mathbb{E}[y_i(1)|i \in T] = \mathbb{E}[y_i(1)]$$

$$\begin{aligned}\mathbb{E}[\Delta_i] &= \mathbb{E}[y_i(1)] - \mathbb{E}[y_i(0)] \\ &= \mathbb{E}[y_i(1)|i \in T] - \mathbb{E}[y_i(0)|i \in C]\end{aligned}$$

Causality in Regression

Set-up

Consider

$$y_i = \mu(0) + \Delta_i D_i + u_i(0)$$
$$D_i^* = \alpha + \beta z_i + v_i$$
$$D_i = \begin{cases} 1 & D^* \geq 0 \\ 0 & D^* < 0 \end{cases}$$

$$\Delta_i = \mu(1) - \mu(0) + u_i(1) + u_i(0)$$
$$= \mathbb{E}[\Delta_i] + u_i(1) - u_i(0)$$

$$\mathbb{E}[u_i(1)] = \mathbb{E}[u_i(0)] = \mathbb{E}[v_i] = 0$$

$$\mathbb{E}[\Delta_i] \equiv \mu(1) - \mu(0) = \text{Common gain for every individual}$$
$$[u_i(1) - u_i(0)] \equiv \text{idiosyncrotic gain that differs for every } i$$

Causality in Regression
ATE and ATT

$$ATE : \mathbb{E}[\Delta_i] = \mu(1) - \mu(0)$$
$$ATT : \mathbb{E}[\Delta_i | D_i = 1] = \mu(1) - \mu(0) + \mathbb{E}[u_i(1) - u_i(0) | D_i = 1]$$

$ATE \neq ATT$ since average idiosyncratic gain for treated $\mathbb{E}[u_i(1) - u_i(0) | D_i = 1]$.

$ATE = ATT$ if

- Idiosyncratic gain zero, $u_i(1) = u_i(0)$, then constant coefficients model.
- Average idiosyncratic gain for treated zero $\mathbb{E}[u_i(1) - u_i(0) | D_i = 1] = 0$, then treatment is random and independent of average idiosyncratic gain

Bias of ATE for random person since $\mathbb{E}[\varepsilon_i \Delta_i] \neq 0$.
Estimated coefficient of y_i on D_i is biased estimate of $\mathbb{E}[\Delta_i]$

Causality in Regression

Biases

$$\mathbb{E}[y_i|D_i = 1] - \mathbb{E}[y_i|D_i = 0] = \mathbb{E}[\Delta_i] + \underbrace{\mathbb{E}[u_i(1) - u_i(0)|D_i = 1] + \mathbb{E}[u_i(0)|D_i = 1] - \mathbb{E}[u_i(0)|D_i = 0]}_{\text{OLS regression bias}}$$

Need controlled experiment so that $\mathbb{E}[u_i(1)] = \mathbb{E}[u_i(1)|D_i = 1] = 0$ and $\mathbb{E}[u_i(0)] = \mathbb{E}[u_i(0)|D_i = 0] = 0$

Same problem for ATT: "Mean Selection Bias" since $\mathbb{E}[\eta_i D_i] \neq 0$.

$$\mathbb{E}[y_i|D_i = 1] - \mathbb{E}[y_i|D_i = 0] = \mathbb{E}[y_i|D_i = 1] + \underbrace{\mathbb{E}[u_i(0)|D_i = 1] - \mathbb{E}[u_i(0)|D_i = 0]}_{MeanSelectionBias}$$

Mean Selection Bias is zero if base state for all the same i.e. $\mathbb{E}[u_i(0)D_i] = 0$

Roy (1951) Model

Idea: Idiosyncratic gain exists and determines treatment participation.

$$\mathbb{E}[D_i | u_i(1) - u_i(0)] \neq \mathbb{E}[D_i]$$

Then

$$\mathbb{E}[u_i(1) - u_i(0) | D_i = 1] \neq \mathbb{E}[u_i(1) - u_i(0)]$$

$$ATE \neq ATT$$

OLS estimator biased for random person

OLS better for treated person, but still mean selection bias