Econometrics

Angrist-Imbens-Rubin Framework

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Notation

Notation

- z is assignment
- D is treatment, $D_i = D_i(z)$
- y is outcome, $y_i = y_i(z, D)$

Three causal effects

- Intention-to-treat effects
  - $z_i \rightarrow D_i$
  - $z_i \rightarrow y_i$
- Treatment effect
  - $D_i \rightarrow y_i$

AIR framework defines assumptions that ensure identification of these effects.

Taxonomy

Figure 1: Compliance Types

		$D_i(Z_i = 0)$	
		0	1
$D_j(Z_j = 1)$	0	NEVER-TAKER $\forall j, D(Z_j) = 0$	DEFIER $\forall i, D(Z_j) = 1 - Z_j$
	1	**COMPLIER** $\forall j, D(Z_j) = Z_j$	ALWAYS-TAKER $\forall j, D(Z_j) = 1$

Source: Fort and Spady (2009) (online at SemanticScholar).

Outline

Assumption #1: SUTVA

Assumption 1:
Stable unit treatment value Assumption (SuTVA)

$$y_i, D_i \perp y_j, D_j, z_j, \qquad i \neq j$$

$$D_i(z) = D_i(z_i)$$
$$y_i(D, z) = y_i(D_i, z_i)$$

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Assumption #2: Random Assignment

Assumption 2:
Random Assignment: $Pr(z_i = 1) = Pr(z_j = 1) \quad i \neq j$

Definition: Causal effect of z_i on D_i: $D_i(1) - D_i(0)$

Definition: Causal effect of z_i on y_i:
$y_i(1, D_i(1)) - y_i(0, D_i(0))$

Under assumption 1 and 2, we can consistently estimate two intention-to-treat average effects:

- $\mathbb{E}[D_i|z_i = 1] - \mathbb{E}[D_i|z_i = 0] = Cov(D_i, z_i)/Var(z_i)$
- $\mathbb{E}[y_i|z_i = 1] - \mathbb{E}[y_i|z_i = 0] = Cov(y_i, z_i)/Var(z_i)$
- Note that the ratio gives the IV estimator

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Assumption #3: Non-Zero Average Causal Effect of Z on D

Assumption 3:
Non-zero average causal effect of z on D

$$Pr(D_i(1) = 1) > Pr(D_i(0) = 1) \leftrightarrow \mathbb{E}[D_i(1) - D_i(0)] \neq 0$$

This requires that assignment to treatment is correlated with treatment indicator (first-stage)

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Assumption #4: Exclusion Restriction

Assumption 4:
Exclusion restriction: z affects y only through D

$$y_i(0, D_i) = y_i(1, D_i) = y_i(D_i)$$

Cannot be observed jointly, so cannot be tested

Given treatment, assignment does not affect outcome

So: $y_i(D_i = 1) - y_i(D_i = 0)$ is causal effect

Assumption #4: Exclusion Restriction

Can now establish effect of z on D and z on y and D on y **at the unit level**.

$$
\begin{aligned}
y_i(1, D_i(1)) - y_i(0, D_i(0)) &= y_i(D_i(1)) - y_i(D_i(0)) \\
&= [y_i(1)D_i(1) + y_i(0)(1 - D_i(1))] \\
&- [y_i(1)D_i(0) + y_i(0)(1 - D_i(0))] \\
&= y_i(D_i(1)) - y_i(D_i(0)) \\
&= [D_i(1) - D_i(0)][y_i(1) - y_i(0)]
\end{aligned}
$$

But cannot identify $\mathbb{E}[y_i(1) - y_i(0)]$ yet

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Assumption # 5: Monotonicity

Assumption 5:
Monotonicity: Exclude defiers $D_i(1) \geq D_i(0)\ \forall i$

ATE (defiers) $= 0$

A.3 and A.5 $\Rightarrow$ Strong monotonicity (at least one complier)

Local Average Treatment Effect (LATE)

LATE is ATE for compliers, i.e. for those who change treatment because of a change in the instrument.

$$LATE \equiv \mathbb{E}[y_i(1) - y_i(0) | D_i(1) - D_i(0) = 1]$$

$$= \frac{\mathbb{E}[y_i(1, D_i(1)) - y_i(0, D_i(0))]}{Pr(D_i(1) - D_i(0) = 1)}$$

Local Average Treatment Effect (LATE)

Alternative Statement

Or:

$$
\begin{aligned}
LATE &\equiv \mathbb{E}[y_i(1) - y_i(0) | D_i(1) = 1, D_i(0) = 0] \\
&= \frac{\mathbb{E}[y_i | z_1 = 1] - \mathbb{E}[y_i | z_i = 0]}{Pr(D_i(1) = 1) - Pr(D_i(0) = 1)} \\
&= \frac{\mathbb{E}[y_i | z_i = 1] - \mathbb{E}[y_i | z_i = 0]}{Pr(D_i = 1 | z_i = 1) - Pr(D_i = 1 | z_i = 0)} \\
&= \frac{Cov(y, z)}{Cov(D, z)}
\end{aligned}
$$

See tables.

Local Average Treatment Effect (LATE)

Causal effect of Z on Y

		$Z_i = 0$	
		$D_i(0) = 0$	$D_i(0) = 1$
$Z_i = 1$	$D_i(1) = 0$	*Never-taker* $Y_i(1,0) - Y_i(0,0) = 0$	*Defier* $Y_i(1,0) - Y_i(0,1) = -(Y_i(1) - Y_i(0))$
	$D_i(1) = 1$	*Complier* $Y_i(1,1) - Y_i(0,0) = Y_i(1) - Y_i(0)$	*Always-taker* $Y_i(1,1) - Y_i(0,1) = 0$

Source: Sascha Becker's econometrics lecture notes.

Local Average Treatment Effect (LATE)

Frequencies

		$Z_i = 0$	
		$D_i(0) = 0$	$D_i(0) = 1$
$Z_i = 1$	$D_i(1) = 0$	*Never-taker* $Pr\{D_i(1) = 0, D_i(0) = 0\}$	*Defier* $Pr\{D_i(1) = 0, D_i(0) = 1\}$
	$D_i(1) = 1$	*Complier* $Pr\{D_i(1) = 1, D_i(0) = 0\}$	*Always-taker* $Pr\{D_i(1) = 1, D_i(0) = 1\}$

Source: Sascha Becker's econometrics lecture notes.

Interpretation

- SUTVA allows to write causal effect for every i independently
- Random assignment allows to estimate LATE using sample statistics
- Exclusion restriction ensures causal effect is zero for always - and never - takers and non-zero only for compliers and defiers (via D)
- Strong monotonicity ensures no defiers and at least one complier
  - LATE is average effect of z on y for compliers
- Denominator of LATE is frequency of compliers, which is also the average causal effect of z on D.
  - LATE - IV estimator is ratio of two intention-to-treat effects.

Critique: LATE vs. Conventional IV

- AIR framework provides assumptions under which IV estimates ATE, not ATT.
- For ATT, IV assumes causal effect same for all treated independently of assignment $\Rightarrow$ effect of D on y same for compliers and always takers.
- IV approach hides assumption of strong monotonicity.
- IV can only identify LATE with these assumptions.
- Critique of IV: Late defined for unobservable sub-population and instrument-dependent.
- LATE difficult in general equilibrium context.
  - LATE unsuitable for interesting policy questions?