Econometrics

Mulit-Response Models

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Multi-Response Models

The Model

$$y_i = \begin{cases} 1 & \text{if } y_i^* < \gamma_1 \\ 2 & \text{if } \gamma_1 \leq y_i^* < \gamma_2 \\ \vdots & \vdots \\ M & \text{if } \gamma_{M-1} \leq y_i^* \end{cases}$$

Multi-Response Models

Ordered Probit

Ordered Probit:

$$
\begin{aligned}
P(y_i = j) &= P(\gamma_{j-1} \leq y_i^* < \gamma_j) \\
&= \Phi(\gamma_j - x_i'\beta) - \Phi(\gamma_{j-1} - x_i'\beta)
\end{aligned}
$$

$$
\log L = \sum_{i=1}^{n} \sum_{j=1}^{J} y_{ij} \log[\Phi(\gamma_j - x_i'\beta) - \Phi(\gamma_{j-1} - x_i'\beta)]
$$

$$
\hat{\varepsilon}_i^G = \frac{\phi(\hat{\gamma}_{j-1} - x_i'\hat{\beta}) - \phi(\hat{\gamma}_j - x_i'\hat{\beta})}{\Phi(\hat{\gamma}_{j-1} - x_i'\hat{\beta}) - \Phi(\hat{\gamma}_j - x_i'\hat{\beta})}
$$

Multi-Response Models

Marginal Effects

Marginal Effects:

- $\frac{\partial P(y_i=j)}{\partial x_i} = [\phi(\gamma_{j-1} - x_i'\beta) - \phi(\gamma_j - x_i'\beta)]\beta$
- $\frac{\partial P(y_i>j)}{\partial x_i} = \phi(\gamma_j - x_i'\beta)\beta$
- Example/Interpretation !

Unordered Alternatives

- Stochastic utility model (XXX: $P(y_i = j)) = \frac{e^{S_{ij}}}{\sum_{j=1}^{M} e^{S_{ij}}}$
- Multinomial Logit

$$\frac{\partial P_{ij}}{\partial x_{ij}} = P_{ij}(1 - P_{ij})\beta$$
$$\frac{\partial P_{ik}}{\partial x_{ij}} = -P_{ij}P_{ik}\beta$$

- Independence from irrelevant alternatives